CAPACITY EXPANSION INVESTMENT PLANNING MODEL

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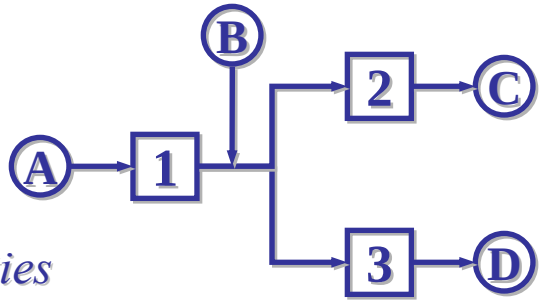
Deterministic Planning

Process Planning Under Uncertainty

GIVEN:

- **Process Network**
  - *Set of Processes*
  - *Set of Chemicals*
- **Forecasted Data**
  - *Demands & Availabilities*
  - *Costs & Prices*
  - *Capital Budget*



DETERMINE:

- **Network Expansions**
  - *Timing*
  - *Sizing*
  - *Location*
- **Production Levels**

OBJECTIVES:

- **Maximize Net Present Value**

Capacity Investment Planning

Design Variables: to be decided before the uncertainty reveals

$$x = \{ Y_{it}, E_{it}, Q_{it} \}$$

Y: Decision of building process *i* in period *t*
E: Capacity expansion of process *i* in period *t*
Q: Total capacity of process *i* in period *t*

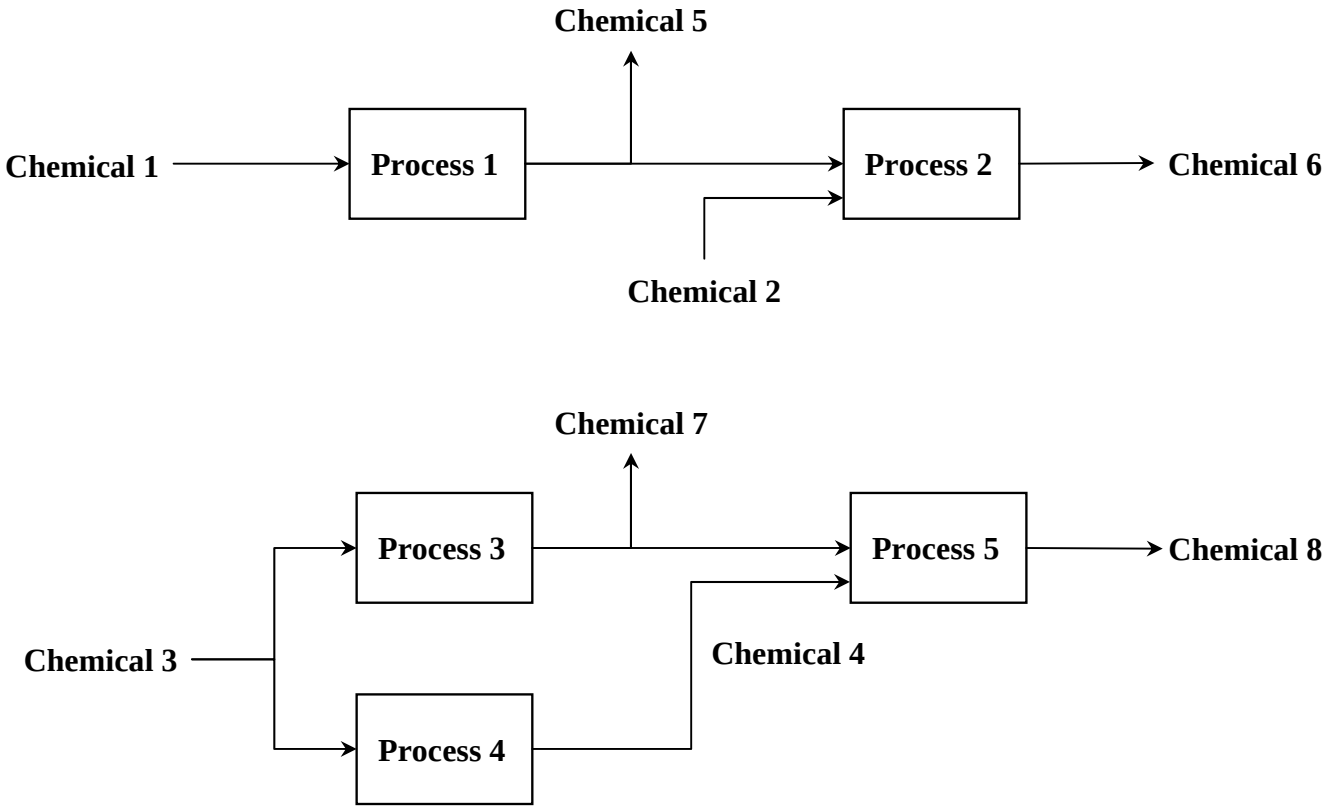
Control Variables: selected after uncertain parameters become known.
Assume them known for the time being!!!!

$$y_s = \{ S_{jlt}, P_{jlt}, W_{it} \}$$

S: Sales of product *j* in market *l* at time *t* and scenario *s*
P: Purchase of raw mat. *j* in market *l* at time *t* and scenario *s*
W: Operating level of of process *i* in period *t* and scenario *s*

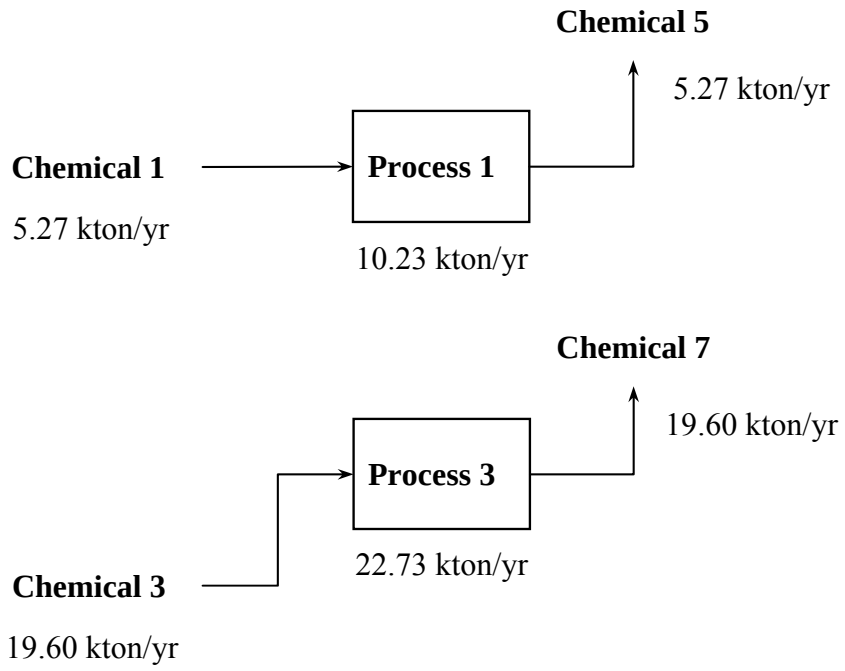
Example

- Project Staged in 3 Time Periods of 2, 2.5, 3.5 years



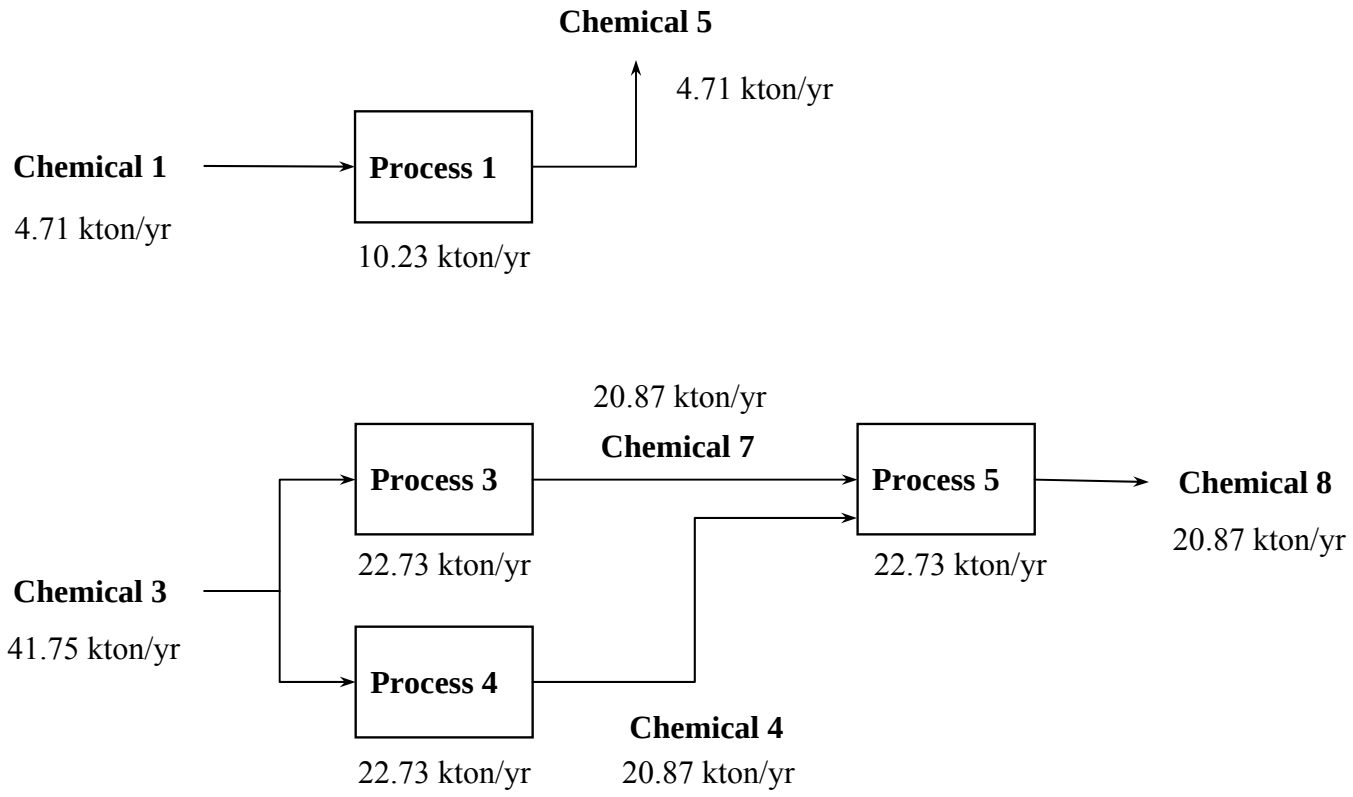
Solution

Period 1
2 years



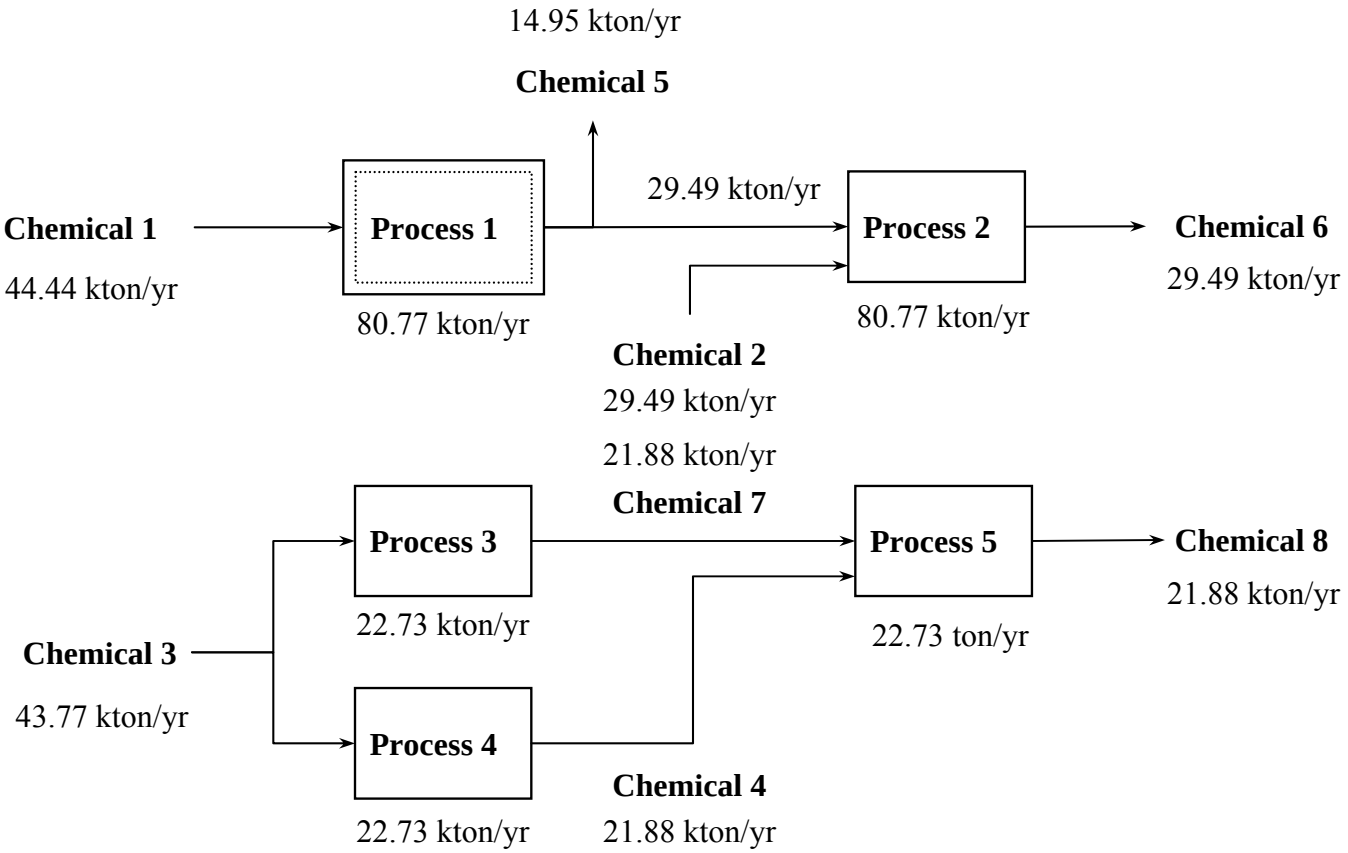
Solution

Period 2
2.5 years



Solution

Period 3
3.5 years



MODEL

SETS

I : Processes i,=1,…,NP
J : Raw materials and Products, j=1,…,NC
T: Time periods. T=1,…,NT
L: Markets, l=1,..NM

VARIABLES

Y_{it}: *An expansion of process I in period t takes place (Yit=1), does not take place (Yit=0)*
E_{it}: *Expansion of capacity of process i in period t.*
Q_{it}: *Capacity of process i in period t.*
W_{it}: *Utilized capacity of process i in period t.*
P_{jlt} : *Amount of raw material/intermediate product j consumed from market l in period t*
S_{jlt} : *Amount of intermediate product/product j sold in market l in period t*

PARAMETERS

η_{ij} : *Amount of raw material/intermediate product j used by process i*
μ_{ij} : *Amount of product/intermediate product j consumed by process i*
γ_{jlt} : *Sale price of product/intermediate product j in market l in period t*
Γ_{jlt} : *Cost of product/intermediate product j in market l in period t*
δ_{it} : *Operating cost of process i in period t*
α_{it} : *Variable cost of expansion for process i in period t*
β_{it} : *Fixed cost of expansion for process i in period t*
L_t : *Discount factor for period t*
E_{it}^L, E_{it}^U : *Lower and upper bounds on a process expansion in period t*
a_{jlt}^L, a_{jlt}^U : *Lower and upper bounds on availability of raw material j in market l in period t*
d_{jlt}^L, d_{jlt}^U : *Lower and upper bounds on demand of product j in market l in period t*
CI_t : *Maximum capital available in period t*
$NEXP_t$: *maximum number of expansions in period t*

MODEL

OBJECTIVE FUNCTION

$$Max\ \ NPV = \underbrace{\sum_{t=1}^{NT} L_t \left(\sum_{l=1}^{NM} \sum_{j=1}^{NC} \left(\gamma_{jlt} S_{jlt} - \Gamma_{jlt} P_{jlt} \right) - \sum_{i=1}^{NP} \delta_{it} W_{it} \right)}_{\text{DISCOUNTED REVENUES}} - \underbrace{\sum_{i=1}^{NP} \sum_{t=1}^{NT} \left(\alpha_{it} E_{it} + \beta_{it} Y_{it} \right)}_{\text{INVESTMENT}}$$

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L_t : *Discount factor for period t*

MODEL

LIMITS ON EXPANSION

$$Y_{it}E_{it}^{L} \le E_{it} \le Y_{it}E_{it}^{L} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

TOTAL CAPACITY IN EACH PERIOD

$$Q_{it} = Q_{i(t-1)} + E_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

LIMIT ON THE NUMBER OF EXPANSIONS

$$\sum_{t=1}^{NT} Y_{it} \le NEXP_i \qquad i=1,\ldots,NP$$

LIMIT ON THE CAPITAL INVESTMENT

$$\sum_{i=1}^{NP} \left(\alpha_{it}E_{it} + \beta_{it}Y_{it}\right) \le CI_t \qquad t=1,\ldots,NT$$

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I : Processes i,=1,…,NP
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L: Markets, l=1,..NM

$NEXP_t$: maximum number of expansions in period t
α_{it} : Variable cost of expansion for process i in period t
β_{it} : Fixed cost of expansion for process i in period t
E_{it}^{L}, E_{it}^{U} : Lower and upper bounds on a process expansion in period t

MODEL

UTILIZED CAPACITY IS LOWER THAN TOTAL CAPACITY

$$W_{it} \le Q_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

MATERIAL BALANCE

$$\sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} \le \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$

BOUNDS

$$a_{jlt}^{L} \le P_{jlt} \le a_{jlt}^{U} \qquad d_{jlt}^{L} \le S_{jlt} \le d_{jlt}^{U} \qquad j=1,\ldots,NC,\ t=1,\ldots,NT,\ l=1,\ldots,NM$$

NONNEGATIVITY $E_{it}, Q_{it}, W_{it}, P_{jlt}, S_{jlt} \ge 0 \qquad \forall i,j,l,t$

INTEGER VARIABLES $Y_{it} \in \{0,1\}$ $\quad i=1,\ldots,NP \quad t=1,\ldots,NT$

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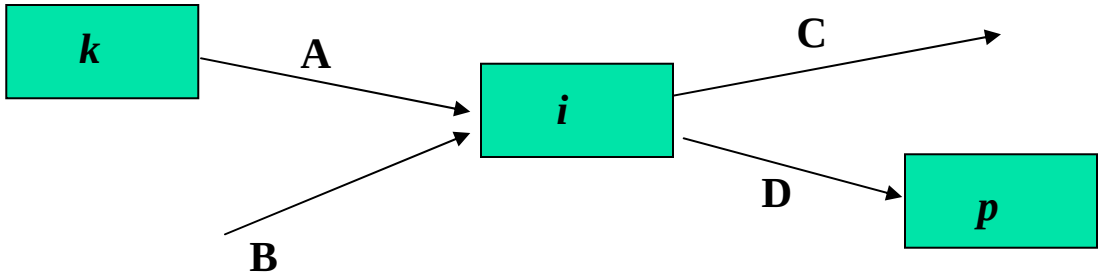
a_{jlt}^{L}, a_{jlt}^{U} : Lower and upper bounds on availability of raw material j in market l in period t

d_{jlt}^{L}, d_{jlt}^{U} : Lower and upper bounds on demand of product j in market l in period t

MODEL

MATERIAL BALANCE

$$\sum_{l=1}^{NM} P_{jlt} + \sum_{i=1}^{NP} \eta_{ij} W_{it} \le \sum_{l=1}^{NM} S_{jlt} + \sum_{i=1}^{NP} \mu_{ij} W_{it} \qquad i=1,\ldots,NP \quad t=1,\ldots,NT$$



$$\sum_{l=1}^{NM} P_{Blt} + \eta_{kA} W_{kt} = \sum_{l=1}^{NM} S_{Clt} + \mu_{iD} W_{it}$$

η_{kA}, μ_{iD}

Reference Component is C

"Stoichiometric" Coefficients